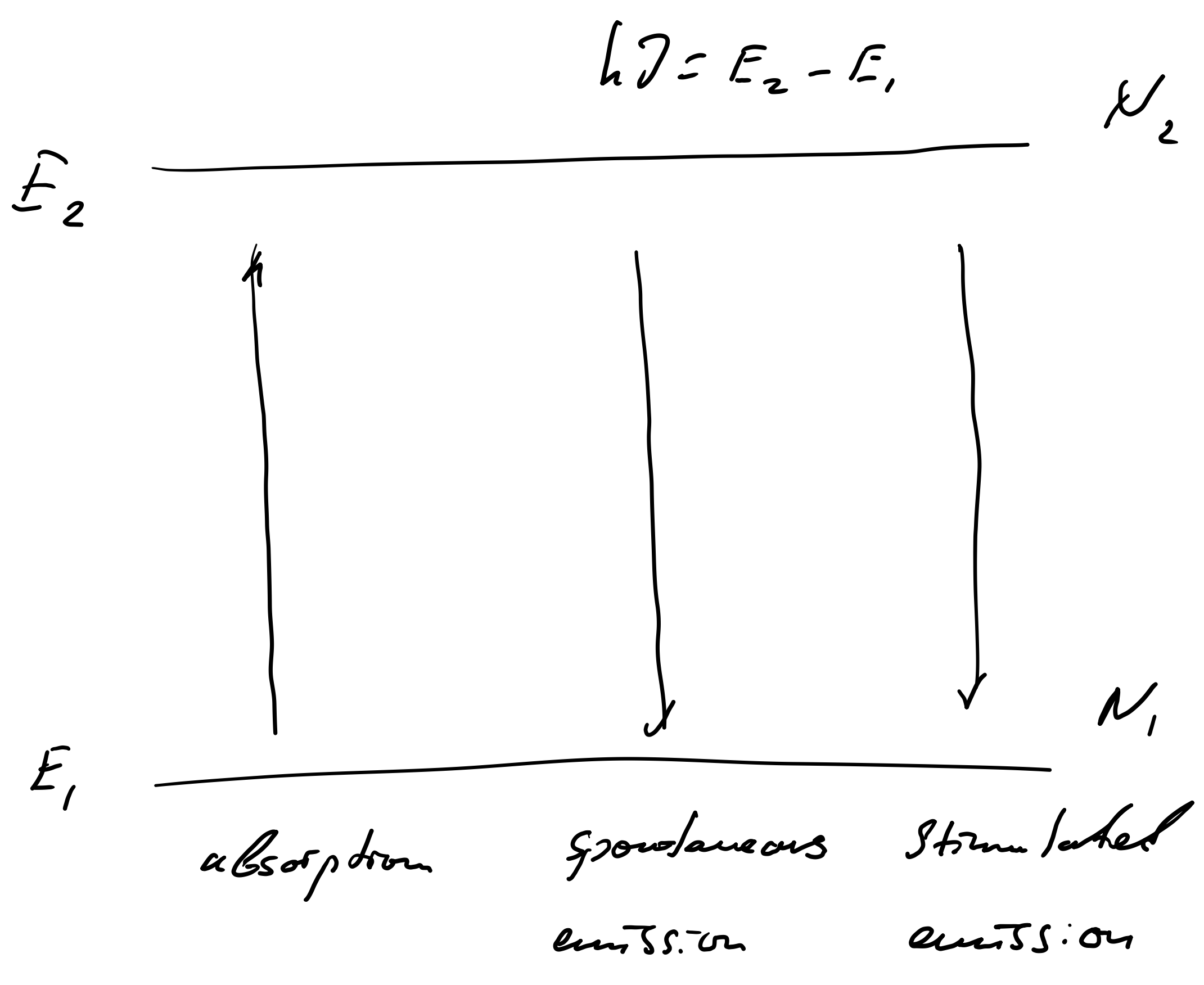
Connection of Einstein coefficients with macroscopic parameters of medium

Let's find connection of Einstein coefficients with macroscopic parameters of the medium.



Since we know

$$\begin{cases} B_{12} = B_{21} \\ A_{21} \to B_{12} \end{cases} \Rightarrow \text{we can find at least one coefficient}$$

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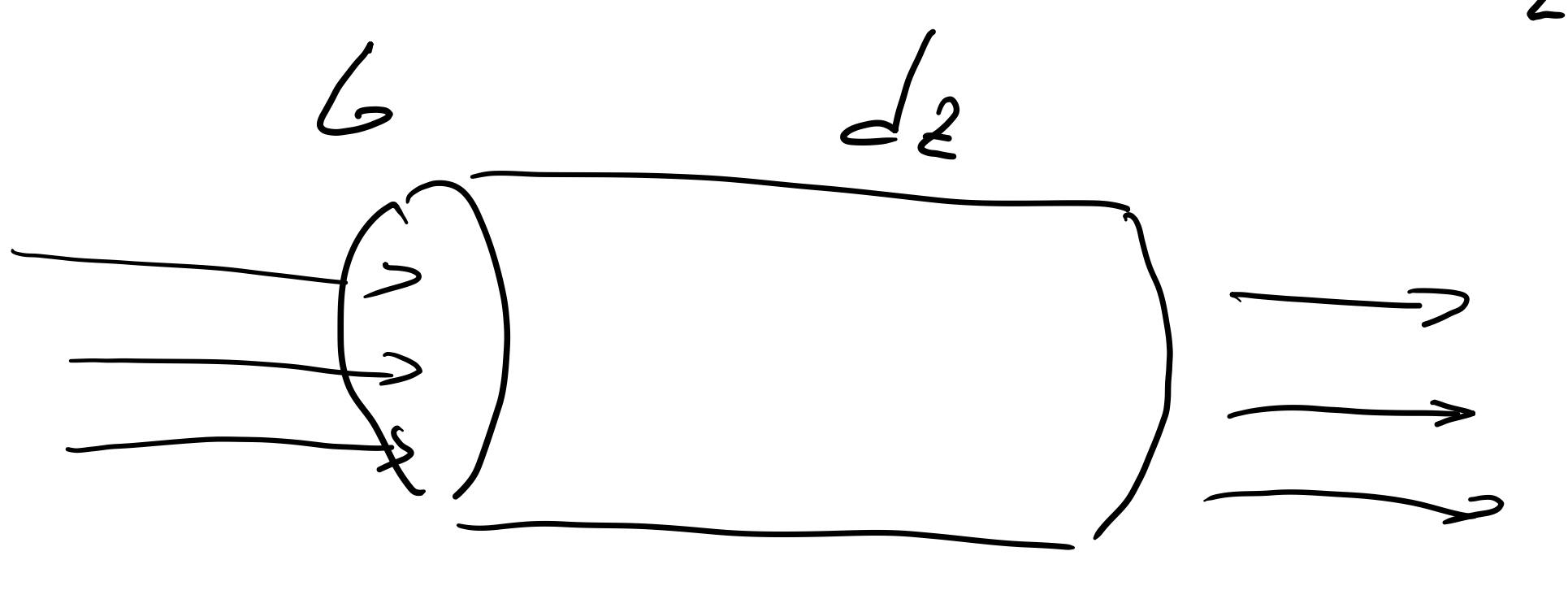


Let's wave of $h\nu$ propagate through media.

We will calculate intensity change when propagate through thickness dz.

Intensity is an energy that is going through elementary surface at unit time.

For this we need to know how many atoms are on level 1 and 2.

$$dV = S\,dz \Rightarrow \begin{cases} N_1\,dV \\ N_2\,dV \end{cases}$$

1) Power of spontaneous emission

$$dP_{sp} = A_{21} N_2\, dV \cdot h\nu$$

dt is 1 second

2) Power of absorption

$$dP_{ab} = B_{12} N_1 u\, dV\, h\nu$$

3) Power of stimulated emission

$$dP_{st} = B_{21} N_2 u\, dV\, h\nu$$

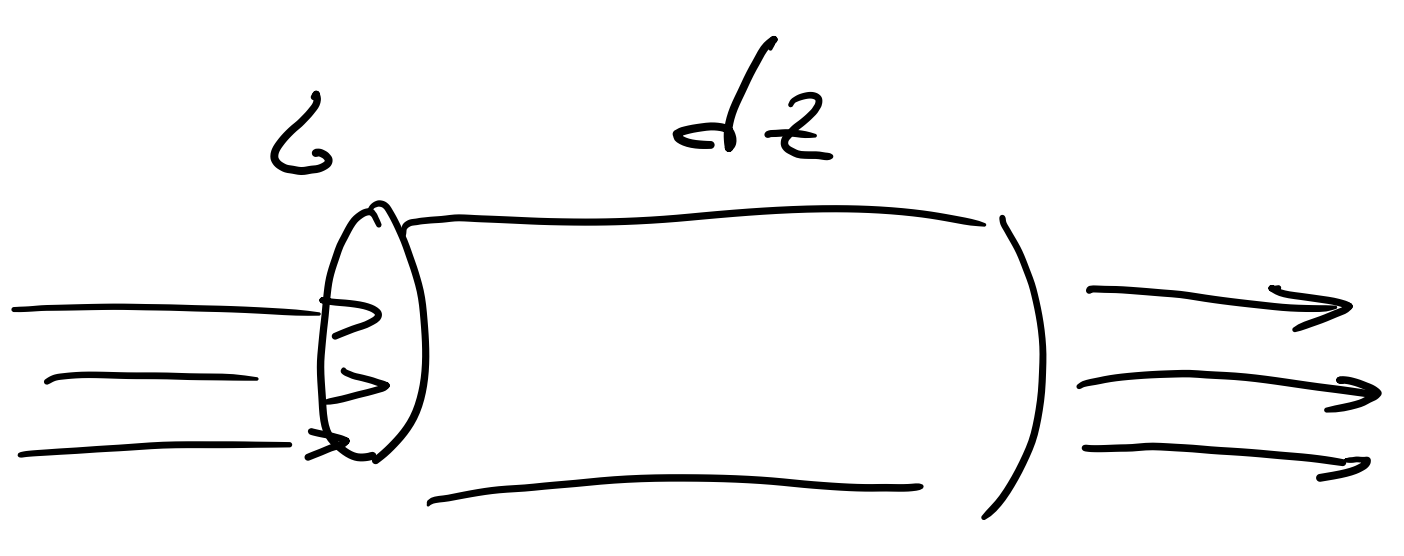
Such processes are going on within our volume $dV = S\,dz$.

Total power in dV:

$$dP = dP_{sp} + dP_{st} - dP_{ab} =$$

($dP_{sp} \to 0$)

We consider plane wave that goes through this volume. Spontaneous photons that are generated inside dV are propagating in random directions. Along z there will be very small amount of spontaneous photons.



For stimulated photon directions conserved due to momentum conservation.

$$dP = B_{12}(N_2 - N_1)\,u\,dV\,h\nu$$

For intensity

$$dI = \frac{dP}{S} = -B_{12}\frac{h\nu}{c}(N_1 - N_2)\underbrace{cu}_{I}\underbrace{\frac{dV}{S}}_{dz} = -\alpha I\,dz$$

$$\boxed{\alpha = B_{12}\frac{h\nu}{c}(N_1 - N_2)}$$

This is quantum perspective of Bouguer-Lambert-Beer law

If $\alpha > 0$, i.e. $N_2 < N_1$, intensity decrease exponentially.

What will happen if we will make $N_2 > N_1$?

$$I = I_0 e^{-\alpha z}$$

E_2 ———

$$\alpha = B_{12}\frac{h\nu}{c}(N_1 - N_2)$$

E_1 ———

$\alpha < 0 \Rightarrow I$ will exponentially grow

$N_2 > N_1$ – population inversion